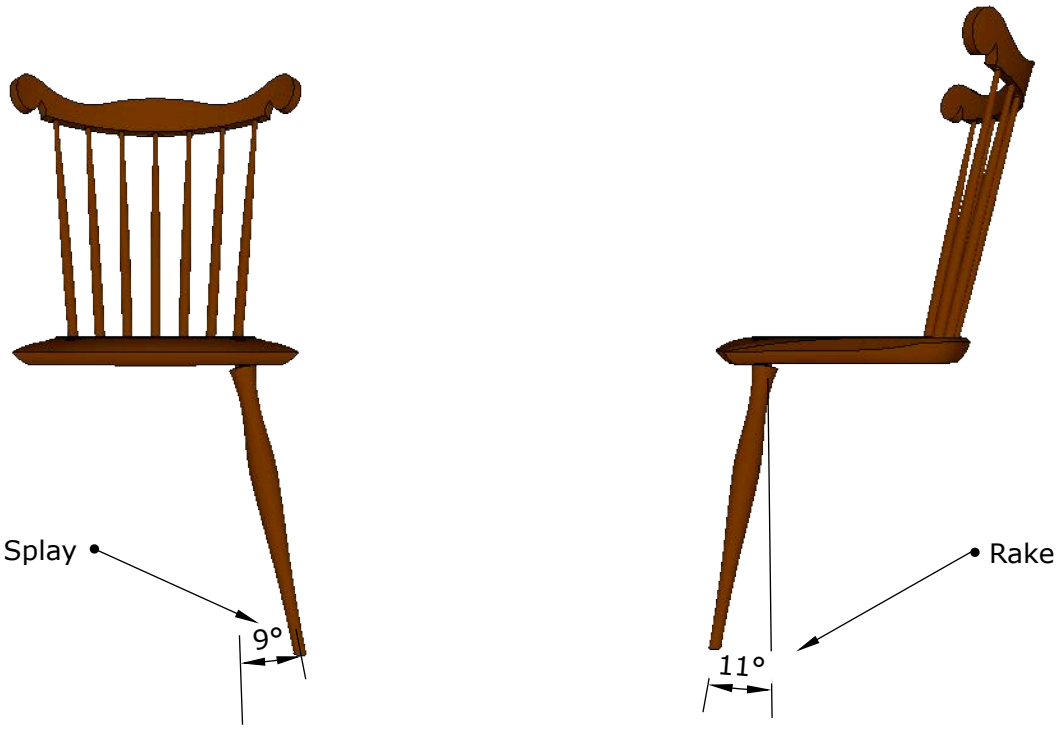
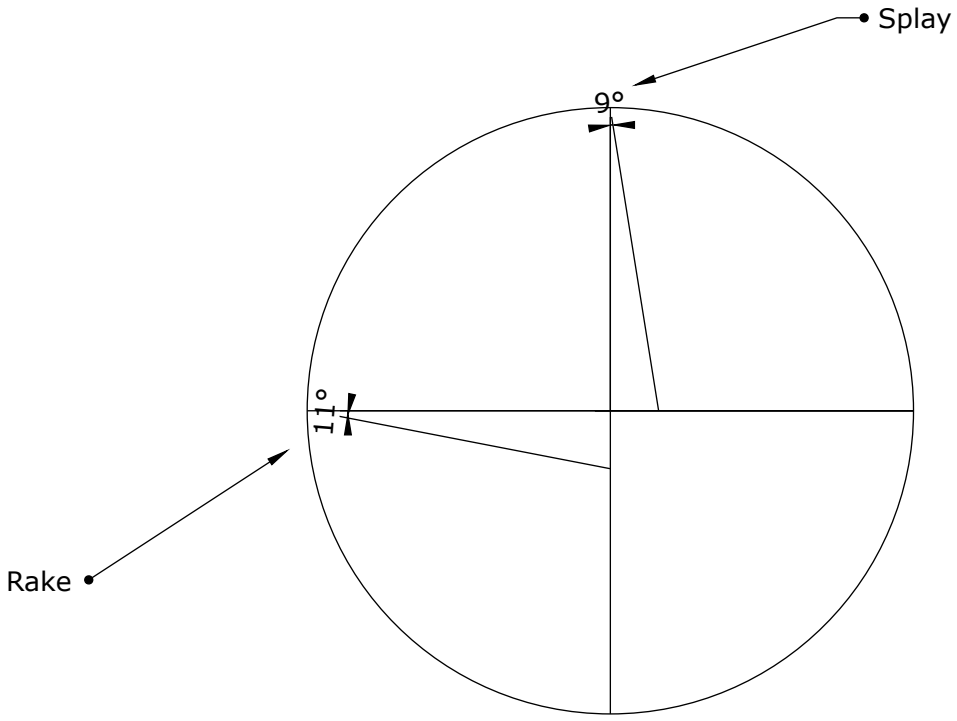
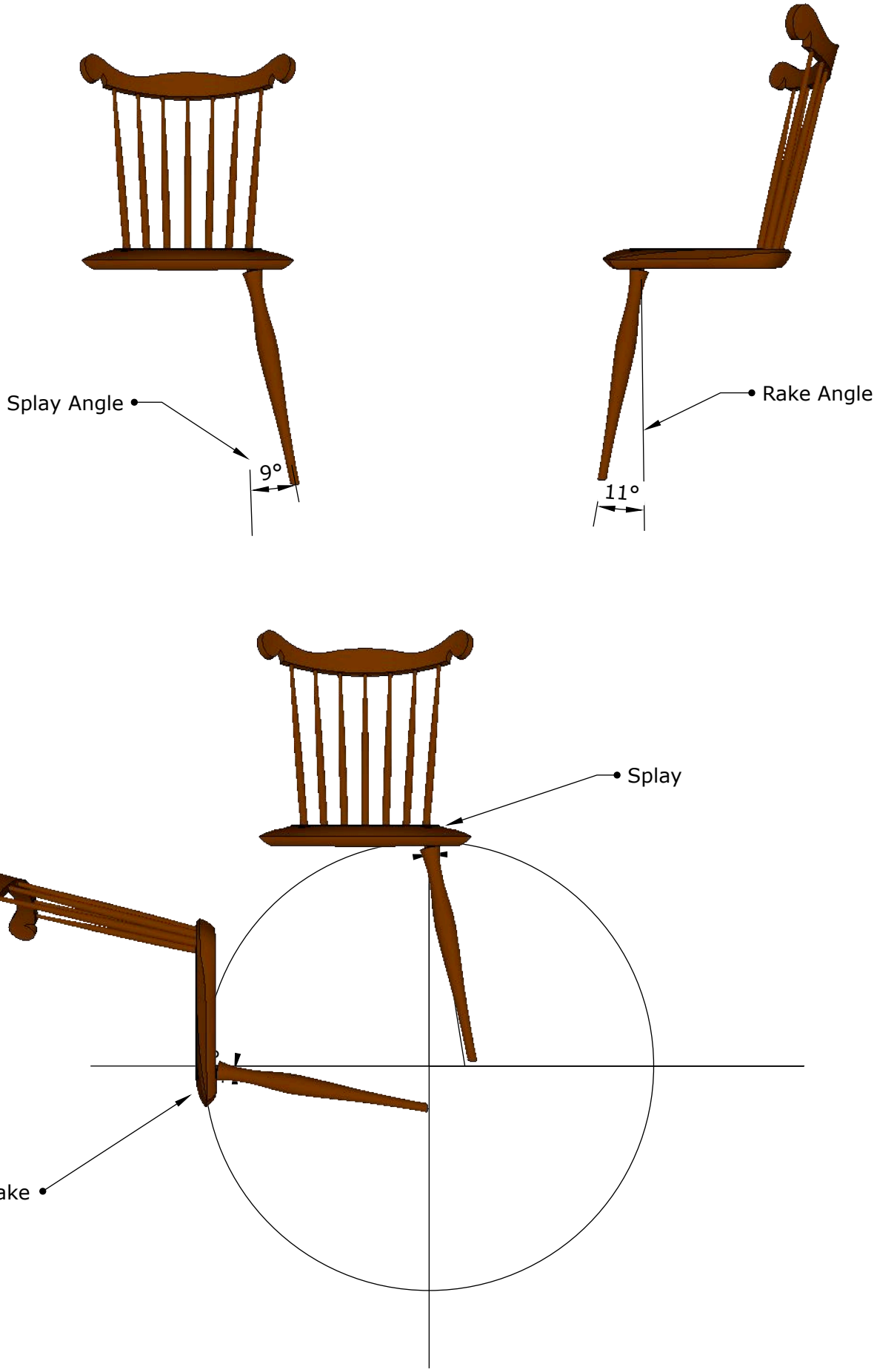
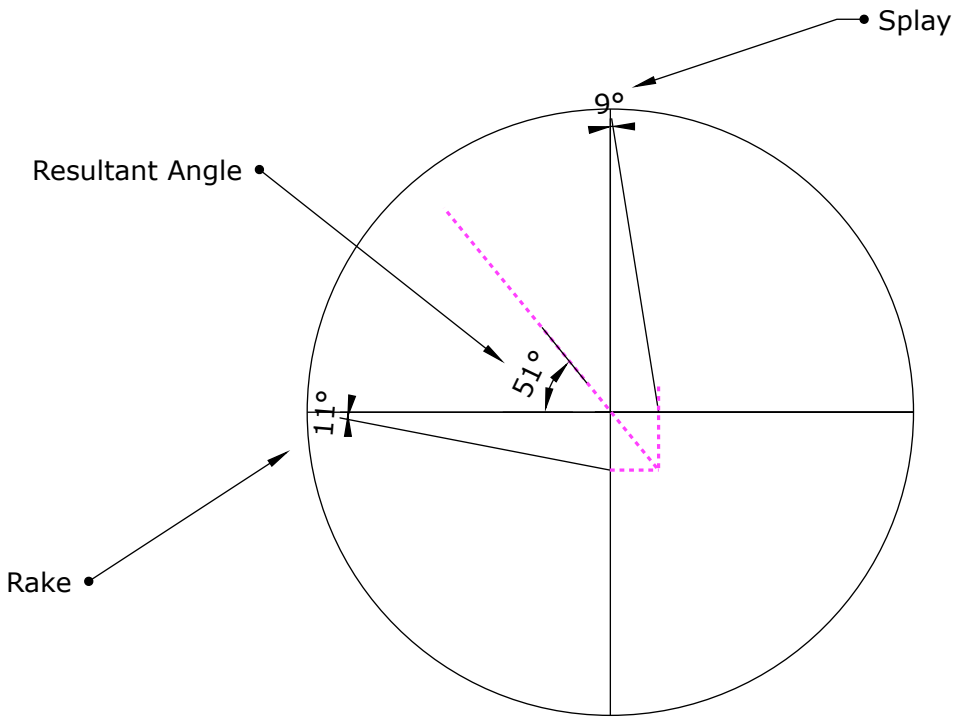
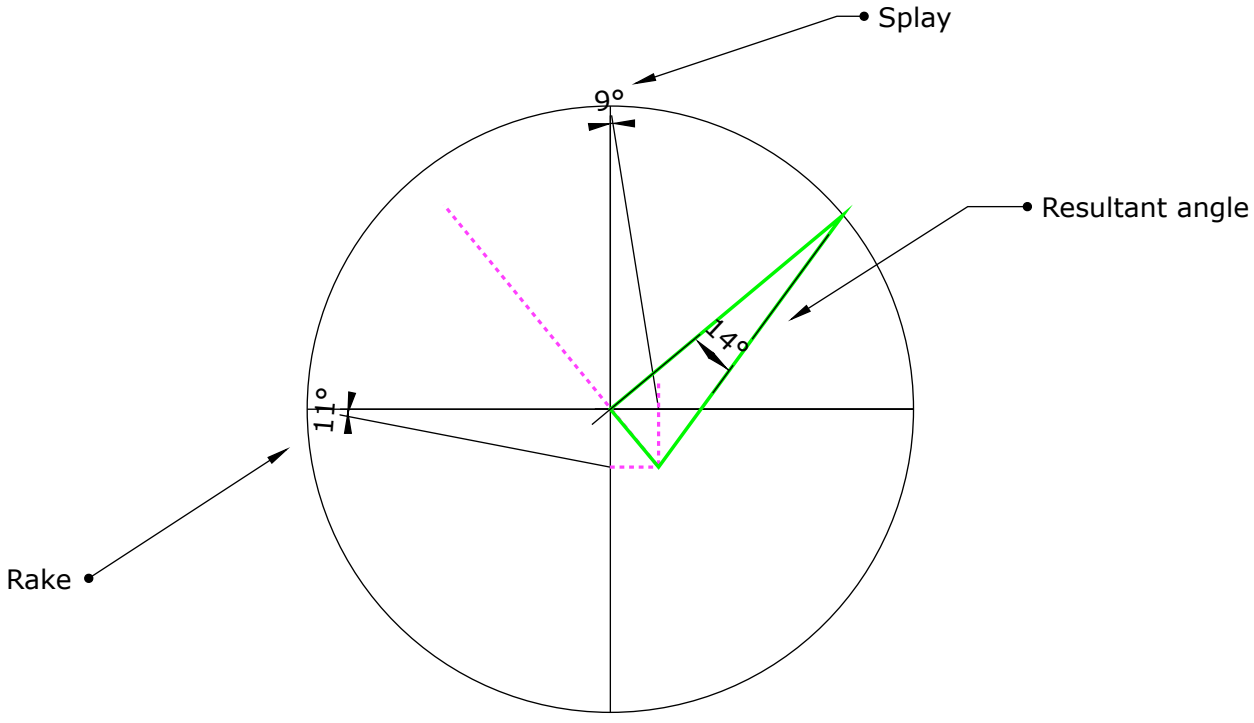
**How to solve using stereotomy (descriptive geometry).**
**This method require no math.**
**Tools needed: Pencil, Paper, compass, straight edge, protractor**

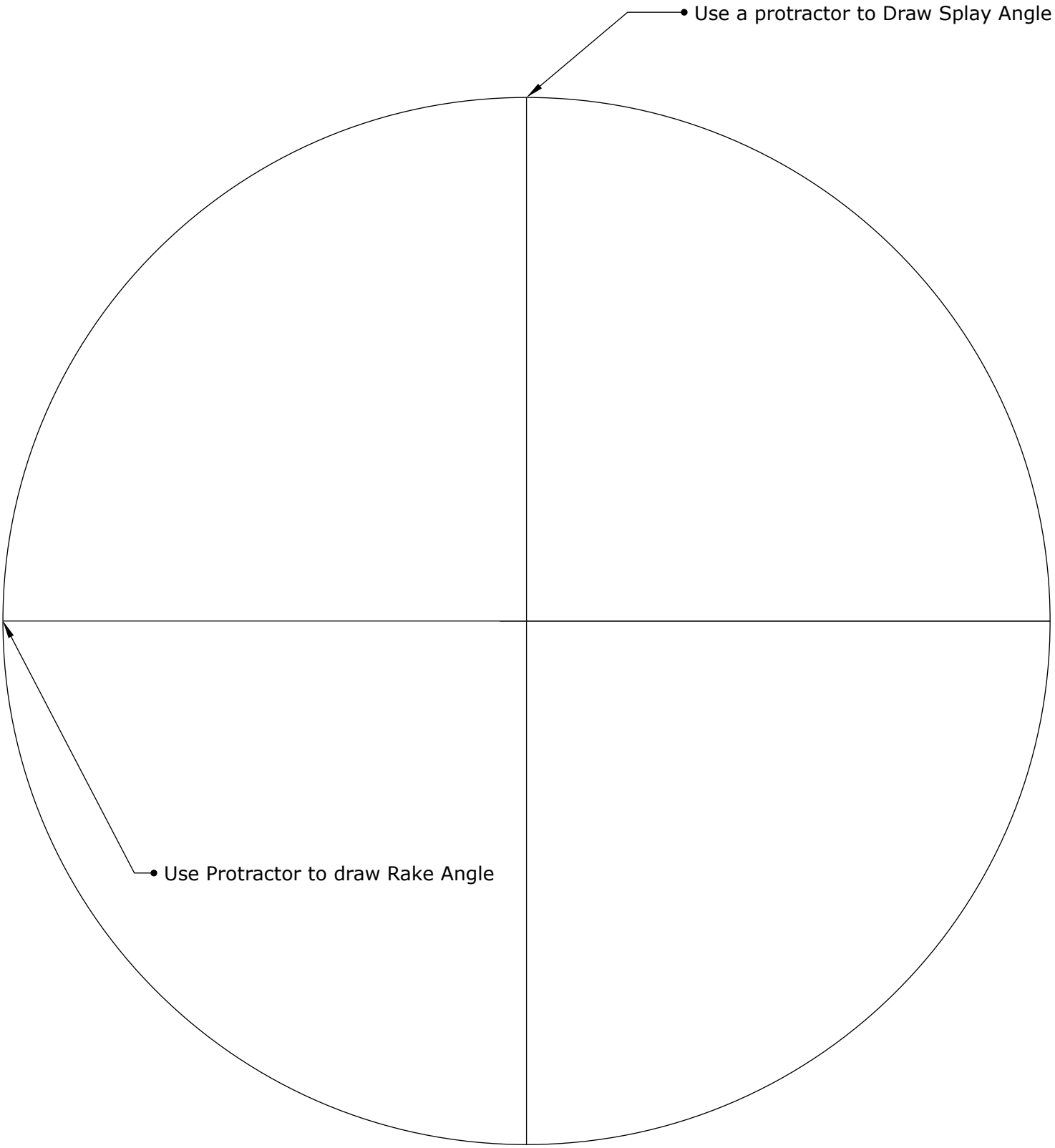


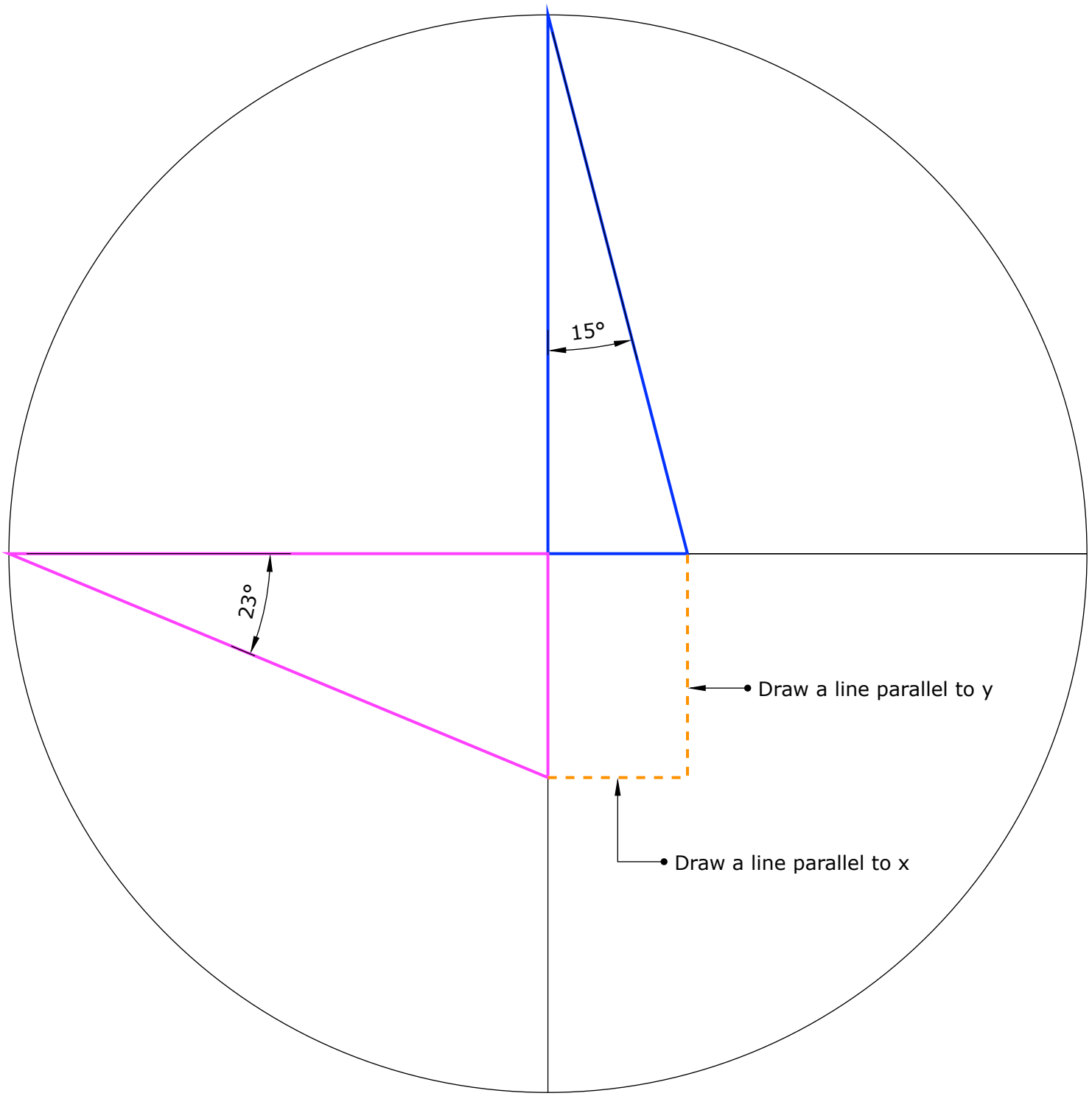



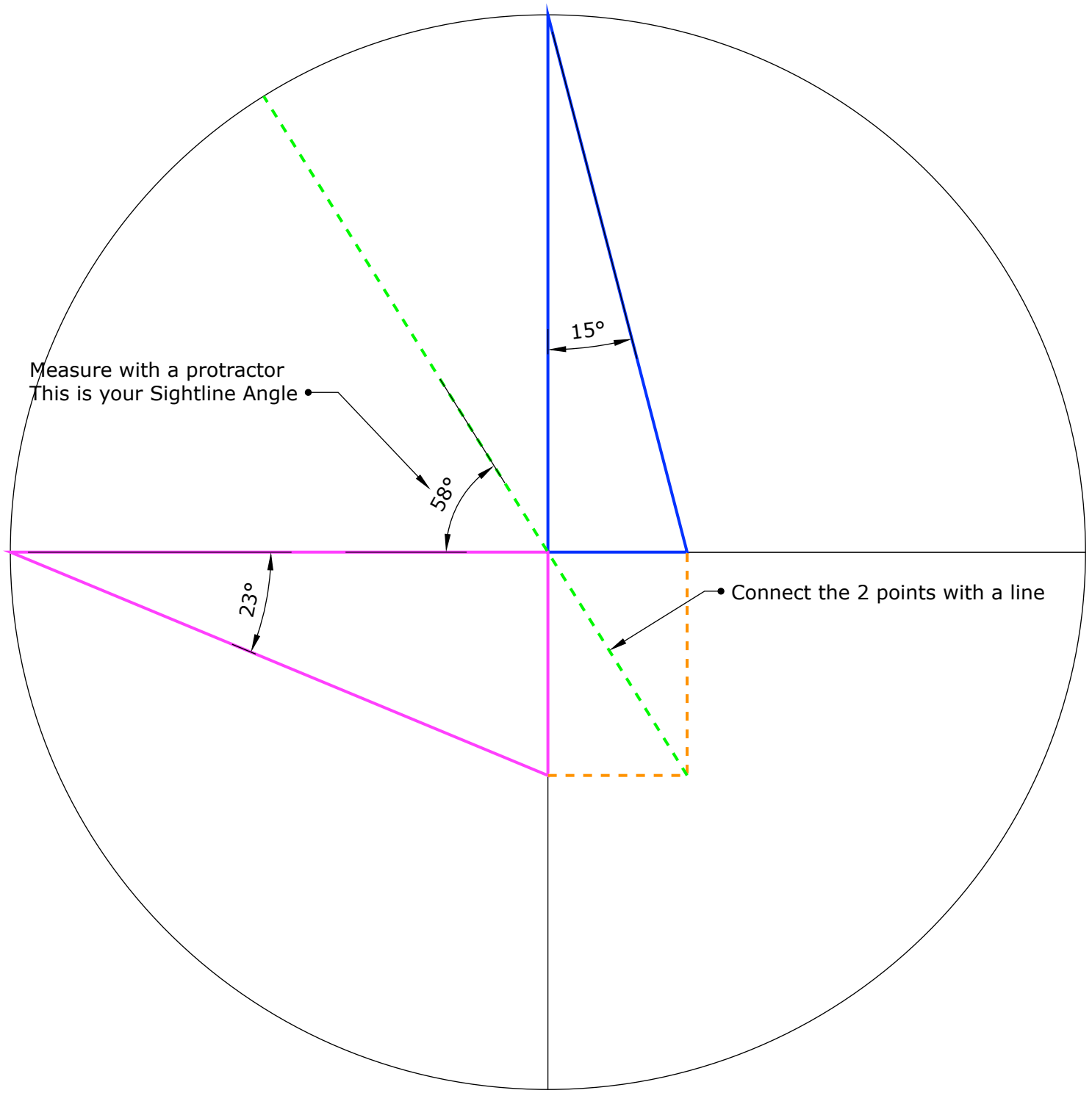

Splay Angle
9°
Rake Angle
11°
Splay
Rake

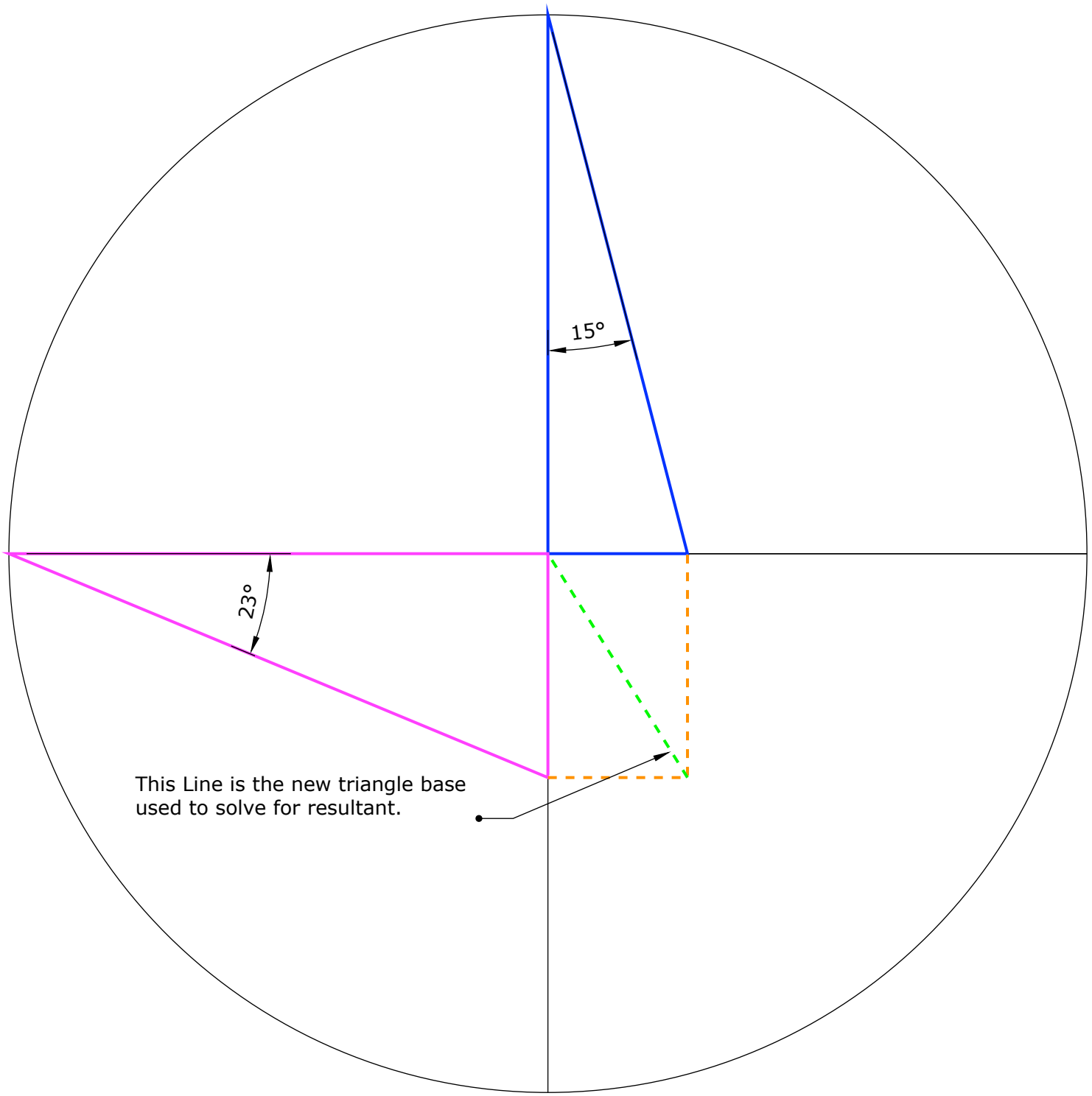

Splay
9°
Resultant Angle
51°
11°
Rake

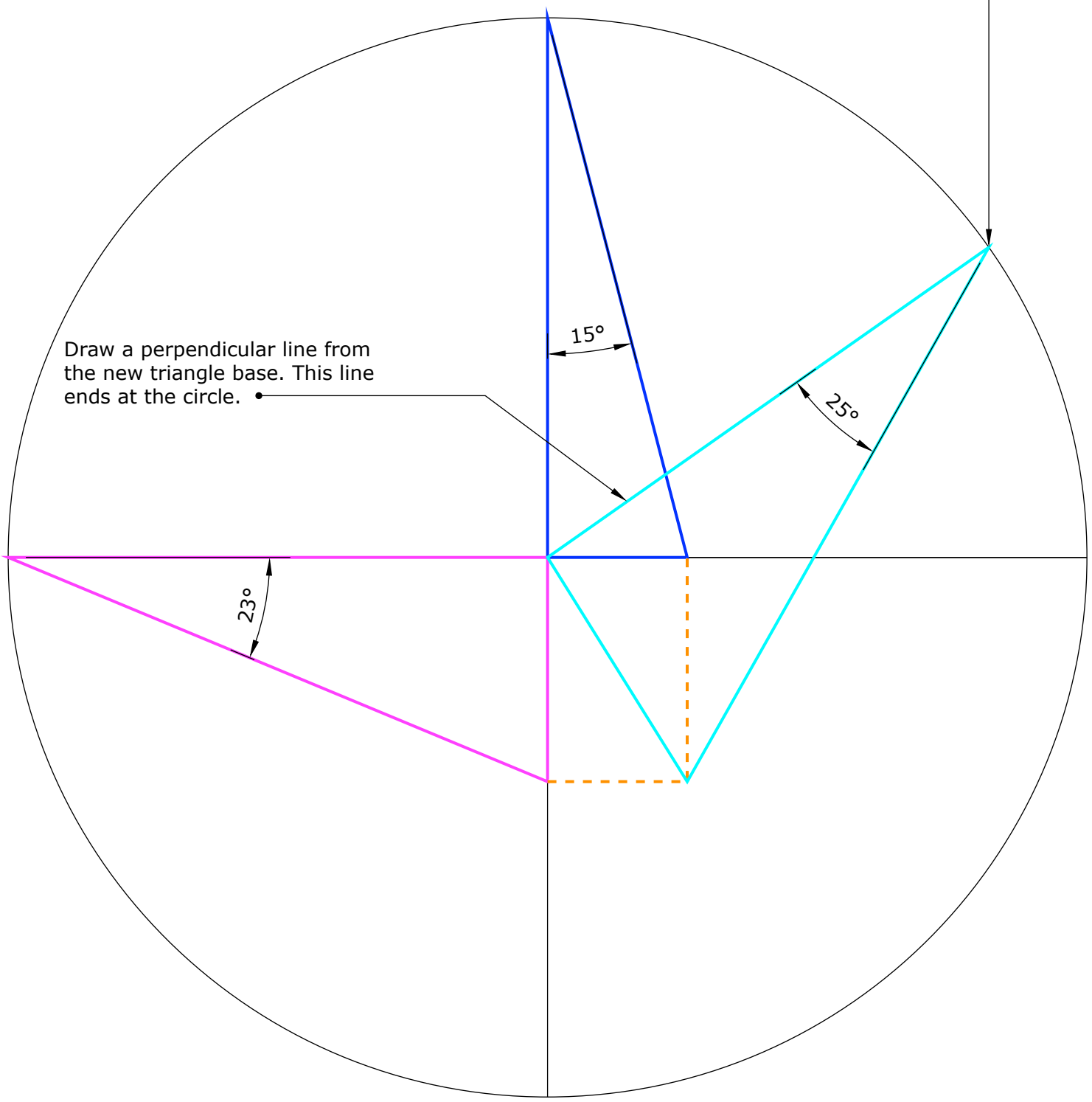

Splay
9°
Resultant angle
14°
11°
Rake


Use a protractor to Draw Splay Angle
Use Protractor to draw Rake Angle


15°
23°
Draw a line parallel to y
Draw a line parallel to x


15°
Measure with a protractor
This is your Sightline Angle
58°
23°
Connect the 2 points with a line


15°
23°
This Line is the new triangle base
used to solve for resultant.


Connect the two points to form the hypotenuse. Measure the angle with a protractor.
THis is you resultant angle.
Draw a perpendicular line from the new triangle base. This line ends at the circle.
15°
25°
23°

Solving for Sightlines and Resultant using Trigonometry

Trigonometry is a branch of mathematics that studies the relationships between the sides and angles of triangles.

In order to have an intuitive understanding of how to solve sightlines and resultant angles (and many other furniture making design problems) you must have an understanding of a few mathematical principles.

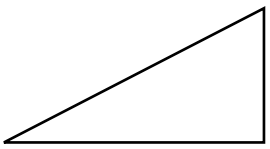
1. Trigonometry and the Tangent ratio function
2. Pythagorean theorem
3. The distribtive property of exponents
4. The Distributive Property

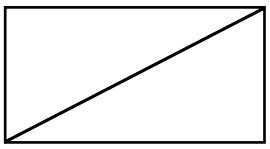
This article is designed to teach you these principles. If you would rather skip the math lesson, you can just use the handouts at the end of this section and follow the sequence of buttons on your calculator to solve the problems.

Trigonometry is a branch of mathematics that studies the relationships between the sides and angles of triangles.

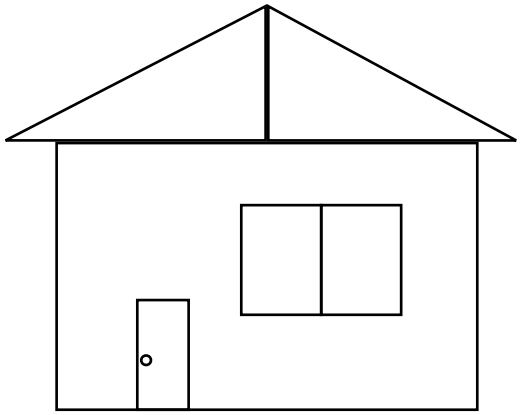
Trigonometric ratio - the ratio between two sides of a right triangle. There are 3 primary "trig ratios" called sine, cosine, and tangent. They are abbreviated sin, cos, and tan respectively. This article will only focus on Tangent. In furniture making design, Timber framing, and roof framing, tangent is by far the most used function. As furniture makers, the tangent is much easier to visualize. It is much simpler to focus on one of these ratios as we learn how to apply them. Once you have a firm understanding of the tangent ration, you will easily be able to apply those skills and use sine and cosine.

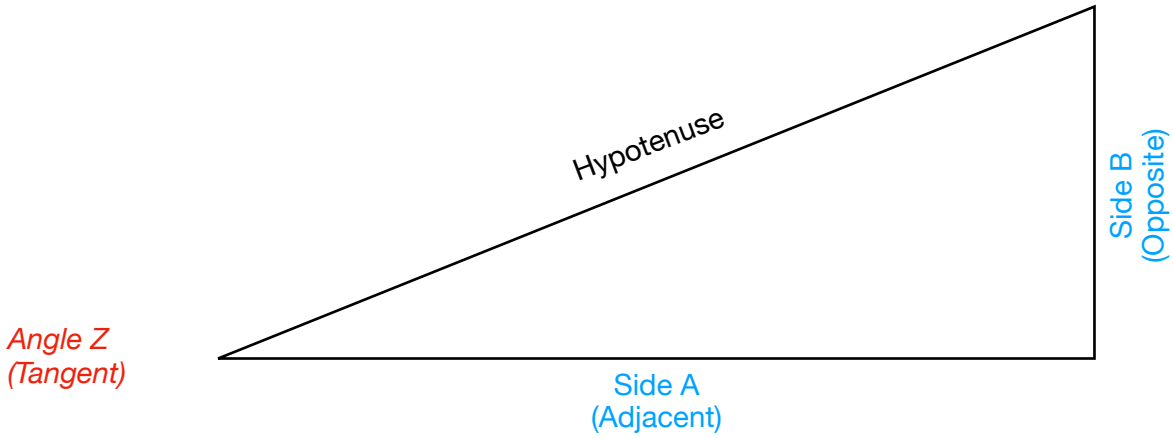
First, lets talk about a triangle. The world is made of triangles. Even a square and rectangle is just two triangles.





Two triangles joined together makes a roof.






While Working with the tangent ratio, we do not need to worry about the hypotenuse. More on solving that later.

Someone very smart figured out that there is a direct relationship between *Angle Z*, Side A, and Side B. If we have any 2 of these variables, we can figure out the third using a math formula. In the above diagram, another name for Side A is the Adjacent side because it is connected to the *Angle Z* that we are trying to solve. Side B is called the opposite because it is on the opposite side of the triangle from *Angle Z*. So we have our angle, Our opposite side, and our adjacent side. The formulas are, Tangent = Opposite/Adjacent and Inverse Tangent = Opposite/Adjacent

$$Tangent = \frac{Opposite}{Adjacent}$$

*We will use this when we want to solve for Side A or Side B.*

$$Tangent^{-}1 = \frac{Opposite}{Adjacent}$$

*We will use this formula when we want to solve for Angle Z.*

Lets looks at this formula in a more familiar way: $2 = \frac{8}{4}$

\

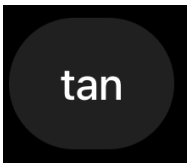
$2 = \frac{8}{4}$ $\qquad 8 = 2 \cdot 4$

*Using some Algebra Skills we can multiply 2x4 to get 8*

$Tangent = \frac{Opposite}{Adjacent}$ $\qquad Opposite = Tangent \cdot Adjacent$

*Since the equation is the same, we can multiply tangent x Adjacent to get the opposite.*

Side Lesson: How to access you scientific calculator on your phone. Your Android or iPhone has a scientific calculator built in. All you have to do is turn your phone sideways. While in this mode, you can access more features by hint the 2nd button or INV on android. Fig. Xxx shows come common buttons that you will need. A very important consideration is that you must have your phone set to degrees and not radians to solve for angles that we recognize as furniture makers and designers. The button to switch modes will be labeled DEG and RAD.




This is the tangent button

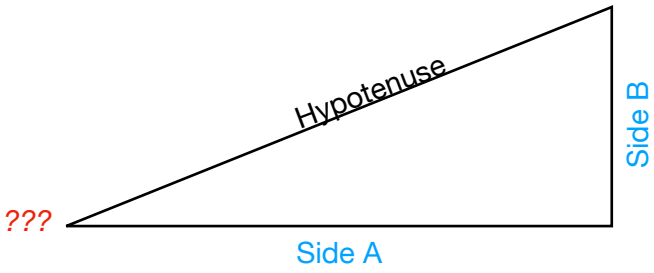

This is the second button. It allows you to access more buttons on the calculator.


This is the inverse tangent button. To find this you must use the 2nd button

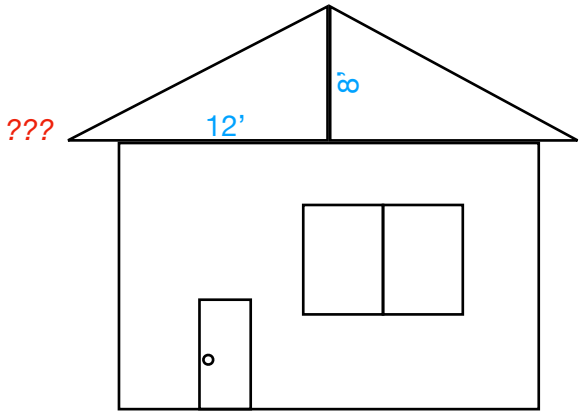

This is the Square Root Key, To find the square root of a number, punch in the number and hit this button.


This is the Square button, To find the square of a number, punch in the number and hit this button.

Now Let's work on some examples.




*Example 1: Solving for angle.*




$$\tan^{-1} = \frac{Opposite}{Adjacent}$$

*If we want to know the angle of the roof, we will use the property of Inverse tangent.*

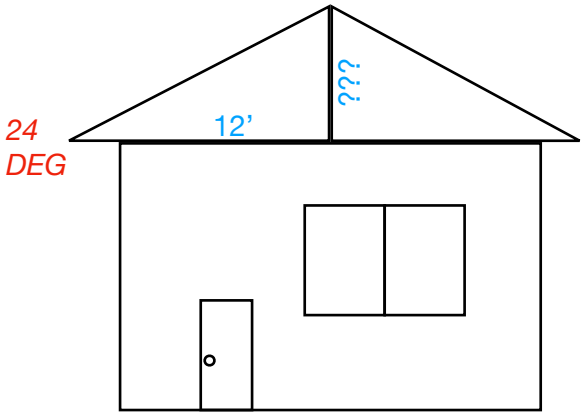
$$\tan^{-1} = \frac{8}{12}$$

*Turn your phone sideways to access the scientific calculator. Type in 8 divided by 12, to get .66666667. Then hit the* 2nd *then* tan⁻¹ *to get 33.69 degrees. Pretty neat right?*

*Most often when solving for an angle, That angle will be represented with a greek letter such as Alpha, beta, theta, etc. For simplicity sake I am going to omit this.*

$$\tan^{-1}\theta = \frac{8}{12}$$

*Example 2: Solving for height of roof.*




$$\tan = \frac{Opposite}{Adjacent}$$

*If we want to know the Height of the roof, we will use the property of Inverse tangent.*

$$24DEG = \frac{???}{12}$$

$$Tangent = \frac{Opposite}{Adjacent} \qquad Opposite = Tangent \cdot Adjacent$$

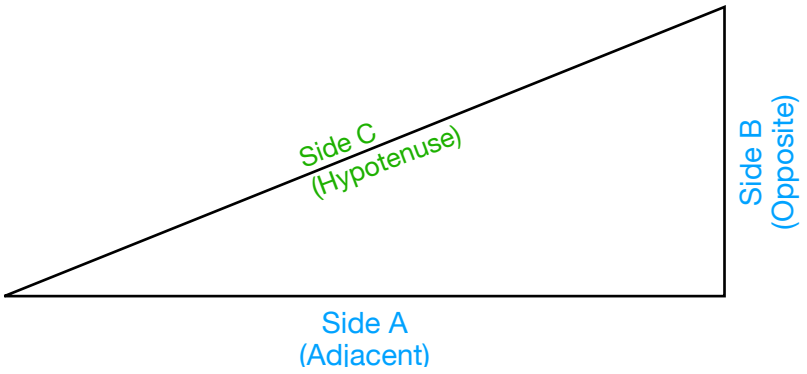
$$24DEG = \frac{???}{12} \qquad ???? = tan(24) \cdot 12$$

*Turn your phone sideways to access the scientific calculator. Type in 24 then hit TAN to get .445228685. Multiply that number by 12 to get 5.342744. That is the height of our roof.*

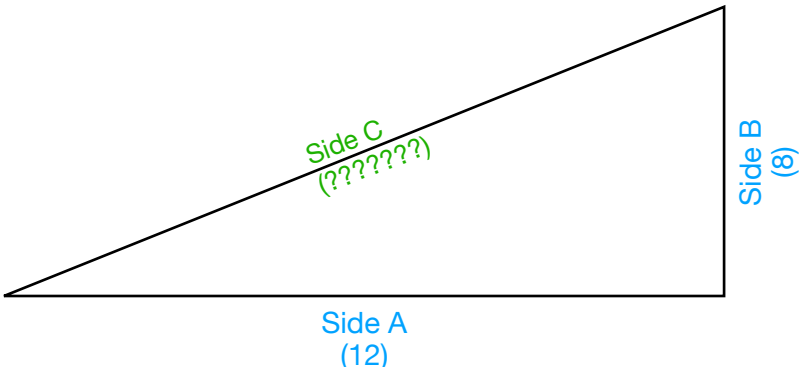
Pythagorean theorem: The **Pythagoras theorem** which is also referred to as the Pythagorean theorem explains the relationship between the three sides of a right-angled triangle. According to the Pythagoras theorem, the square of the hypotenuse is equal to the sum of the squares of the other two sides of a triangle. Let us learn more about the Pythagoras theorem, its derivations, and equations followed by solved examples on the Pythagoras theorem triangle and squares.

$$a^2 + b^2 = c^2 \qquad \text{Or} \qquad c = \sqrt{a^2 + b^2}$$

The Pythagoras theorem equation is expressed as, $c^2 = a^2 + b^2$, where 'c' = hypotenuse of the right triangle and 'a' and 'b' are the other two legs. Hence, any triangle with one angle equal to 90 degrees produces a Pythagoras triangle and the Pythagoras equation can be applied in the triangle




$$c = \sqrt{a^2 + b^2}$$




$$? = \sqrt{12^2 + 8^2}$$

*Turn your phone sideways to access the scientific calculator. Type in 12 then hit* $x^2$ *Then hit the plus button, type in 8 then hit* $x^2$ *again, then hit equals. You will get 208. The SqRT of 208 is the hypotenuse of your triangle. Hit* $\sqrt[2]{x}$ *to get 14.4222. This is the Length of your hypotenuse.*

Congrats, You have learned the geometry of triangles using tangent ratios and Pythagorus theorem. Having a firm understanding of these geometric principles will help you to solve all kinds of problems in woodworking. Lastly, I want to share a little bit of algebra that will strengthen your mathematical skill set for wood design by allowing you to manipulate these formulas.

**Distributive Property: The distributive property tells us how to solve expressions in the form of a(b + c). The distributive property is sometimes called the distributive law of multiplication and division.**

$$a(b+c) = ab + ac \qquad\qquad 8(2+4) = 8 \cdot 2 + 8 \cdot 4$$

*48* *48*

*A more complex version that we will use later.*

$$H^2(\tan^2\theta + \tan^2\beta) = (H^2 \cdot \tan^2\theta) + (H^2 \cdot \tan^2\beta)$$

**The Distributive Property of Exponents: This is yet another way to manipulate formulas.**

$$(ab)^2 = a^2 \cdot b^2 \qquad\qquad (3 \cdot 5)^2 = 3^2 \cdot 5^2$$

$$(15)^2 = 9 \cdot 25$$

*225* *225*

*A more complex version that we will use later.*

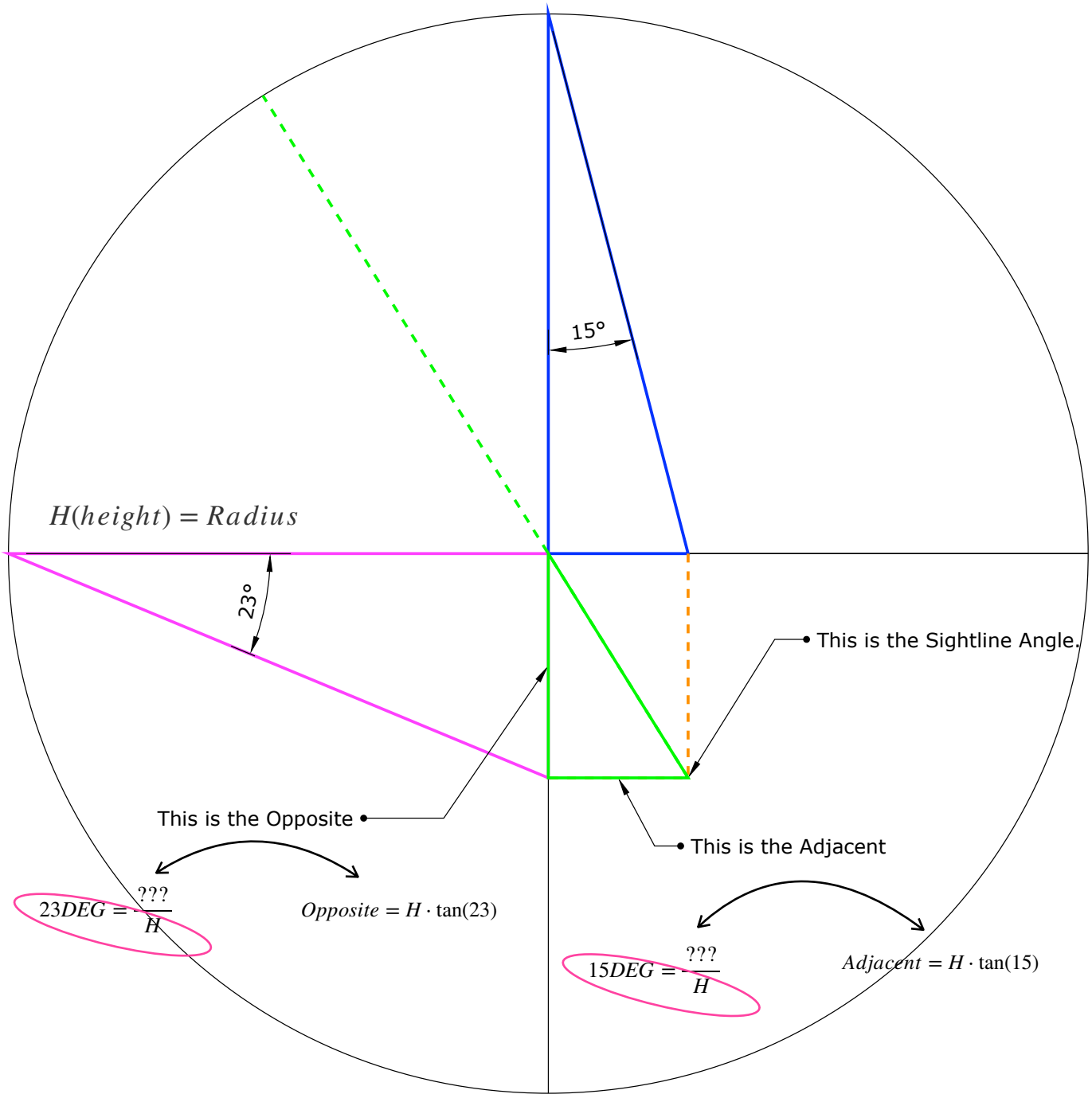
$$(H \cdot \tan\theta)^2 = H^2 \cdot \tan^2\theta$$

# Solve the sightline angle.

$$\tan^{-1} = \frac{Opposite}{Adjacent}$$

$$\tan^{-1} = \frac{H \cdot \tan(rake)}{H \cdot \tan(splay)} \rightarrow \tan^{-1} = \frac{\tan(rake)}{\tan(splay)} \rightarrow \tan^{-1} = \frac{\tan(23)}{\tan(15)}$$

*The H cancels*



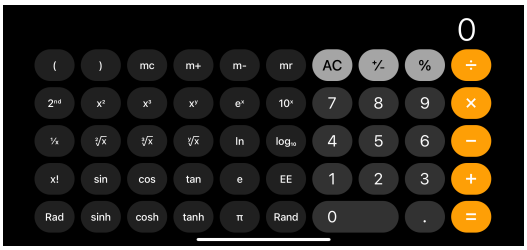
HOW TO SOLVE FOR Sightline ANGLE

Write your rake and splay below

Rake:_____ Splay:_____

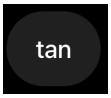
*In the lower right-hand corner, I have included a snippet from one of Curtis Buchanans plans to use for proof. Use these rake and splay numbers to get the hang of it.*

Tools required: Cell phone. *Image from iPhone calculator*



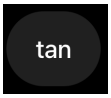

Step 1: Open scientific calculator on phone. For I-phone, turn phone sideways to access scientific mode

Step 2: Punch in your rake number

Step 3: Hit Tangent 


Step 4: Hit Divide 


Step 5: Punch in your splay Number

Step 6: Hit Tangent 


Step 7: Hit equals 


Step 8: Hit the 2nd Button 


Step 9: Hit the Inverse Tangent Button. 


**!!!!!!Please Note: Your phone has 2 angle solving modes, Degree & Radian. You must ensure that your phone is in degree mode to solve. The button is located in the lower left corner. It is labeled DEG or RAD. Press this button to switch back and forth. You are in the correct mode when the button is labeled RAD. !!!!!!**

| | |
|---|---|
| **Arm Support** | |
| Rake | 19° |
| Splay | 17° |
| Resultant | 24.7° |
| Sighting | 48.4° |
| **Front Leg** | |
| Rake | 7° |
| Splay | 15° |
| Resultant | 16.4° |
| Sighting | 24.6° |
| **Rear Leg** | |
| Rake | 22° |
| Splay | 11° |
| Resultant | 24.2° |
| Sighting | 64.3° |
| **Post** | |
| Rake | 12° |
| Splay | 8° |
| Resultant | 14.3° |
| Sighting | 56.5° |

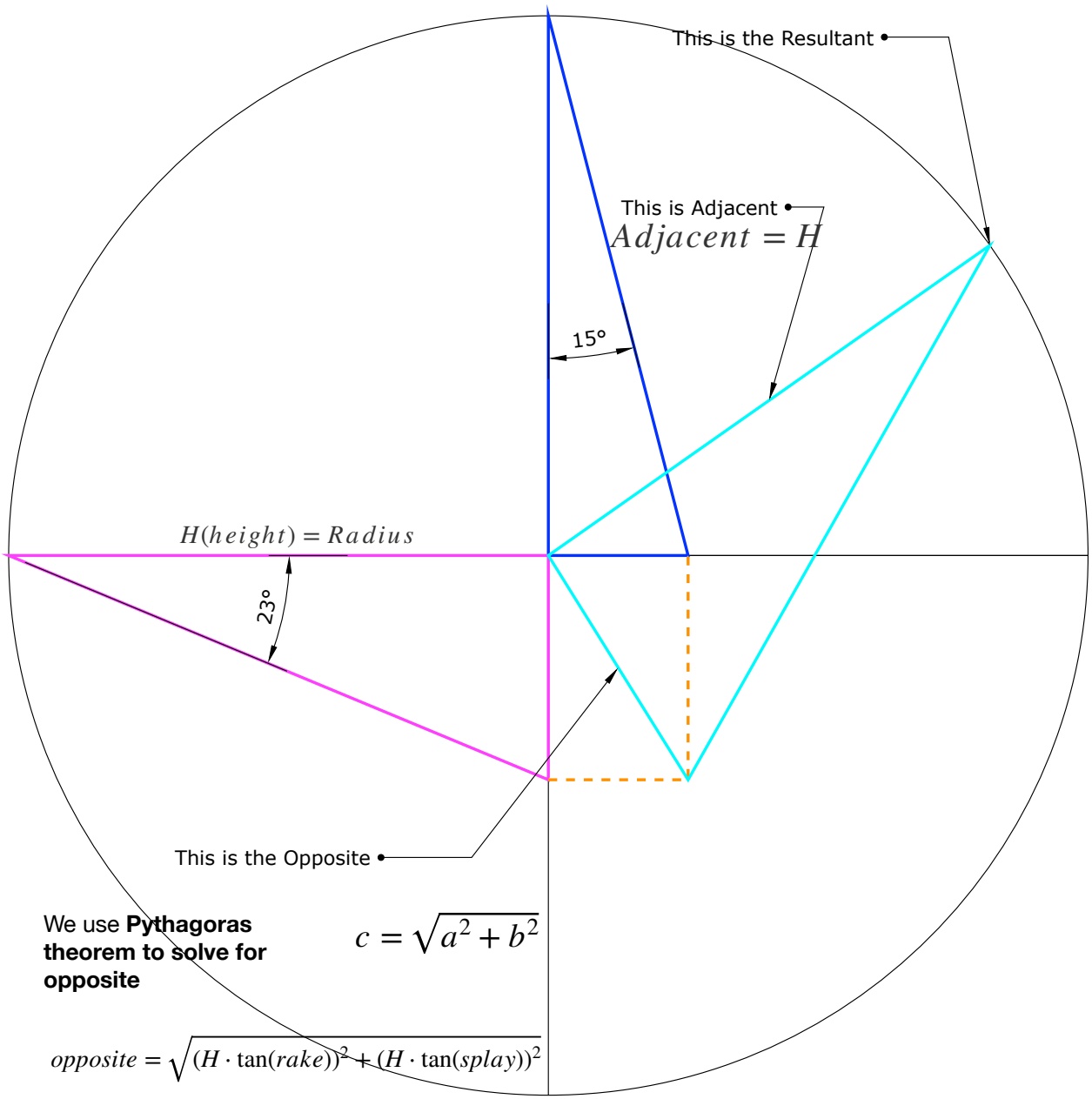
You are done. Write your sightline angle here:_____

# Solve the resultant angle

$$\tan^{-1} = \frac{Opposite}{Adjacent}$$

$$\tan^{-1} = \frac{\sqrt{(h \cdot \tan(rake))^2 + (h \cdot \tan(splay))^2}}{H} \longrightarrow \tan^{-1} = \sqrt{(\tan^2(23) + \tan^2(15)}$$

*See the next page for reduction method used*



This is the process of simplifying the resultant angle formula.

$$\tan^{-1} = \frac{\sqrt{(H \cdot \tan(rake))^2 + (H \cdot \tan(splay))^2}}{H}$$

*Tangent= Opposite/Adjacent*

$$\tan^{-1} = \frac{\sqrt{(H^2 \cdot \tan^2(rake)) + (H^2 \cdot \tan^2(splay))}}{H}$$

*Use distributive property of exponents to distribute the square*

$$\tan^{-1} = \frac{\sqrt{H^2 \cdot (\tan^2(rake) + \tan^2(splay))}}{H}$$

*Use distributive property to move H*

$$\tan^{-1} = \frac{H\sqrt{(\tan^2(rake) + \tan^2(splay))}}{H}$$

*SQRT of H squared is just H. We can move H to the left.*

$$\tan^{-1} = \sqrt{(\tan^2(rake) + \tan^2(splay))}$$

*H cancels out*

$$\tan^{-1} = \sqrt{(\tan^2(23) + \tan^2(15))}$$

*Punch in the Rake and Splay*

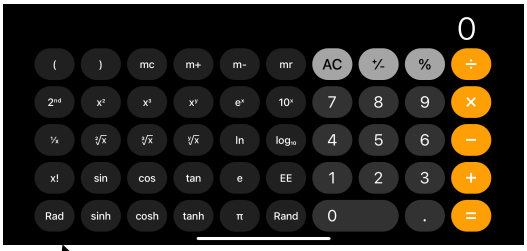
HOW TO SOLVE FOR Resultant ANGLE

Write your rake and splay below

Rake:_____ Splay:_____

*In the lower right-hand corner, I have included a snippet from one of Curtis Buchanans plans to use for proof. Use these rake and splay numbers to get the hang of it.*

Tools required: Cell phone. *Image from iPhone calculator*




Step 1: Open the scientific calculator on phone. For I-phone, turn phone sideways to access scientific mode

Step 2: Punch in your rake number

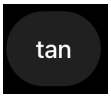
Step 3: Hit Tangent 


Step 4: Hit Squared 


Step 5: Hit Plus 


**Please Note: Your phone has 2 angle solving modes, Degree & Radian. You must ensure that your phone is in degree mode to solve. The button is located in the lower left corner. It is labeled DEG or RAD. Press this button to switch back and forth. You are in the correct mode when the button is labeled RAD.**

Step 6: Punch in your splay Number

Step 7: Hit Tangent 


Step 8: Hit Squared 


Step 9: Hit equals 


Step 10: Hit Square root 


Step 8: Hit the 2nd Button 


Step 9: Hit the Inverse Tangent Button. 


Your Done! Resultant angle:_____

| | |
|---|---|
| **Arm Support** | |
| Rake | 19° |
| Splay | 17° |
| Resultant | 24.7° |
| Sighting | 48.4° |
| **Front Leg** | |
| Rake | 7° |
| Splay | 15° |
| Resultant | 16.4° |
| Sighting | 24.6° |
| **Rear Leg** | |
| Rake | 22° |
| Splay | 11° |
| Resultant | 24.2° |
| Sighting | 64.3° |
| **Post** | |
| Rake | 12° |
| Splay | 8° |
| Resultant | 14.3° |
| Sighting | 56.5° |